The principle of reflection zone plate.

A schematic diagram of an off-axis reflection zone plate (RZP) imprinted as a projection on a totally reflecting mirror surface is shown in figure 1 [1,2]. The structure, being a two-dimensional laminar grating of variable line spacing located at a distance R_1 from a source, is capable of imaging the source by diffraction onto a certain distance R_2 along the optical axis; acting as both a dispersive and focusing optical element. However, due to the high chromaticity of a zone plate – the dependence of the focal length on wavelength – different energies are focused at different positions along the optical axis.

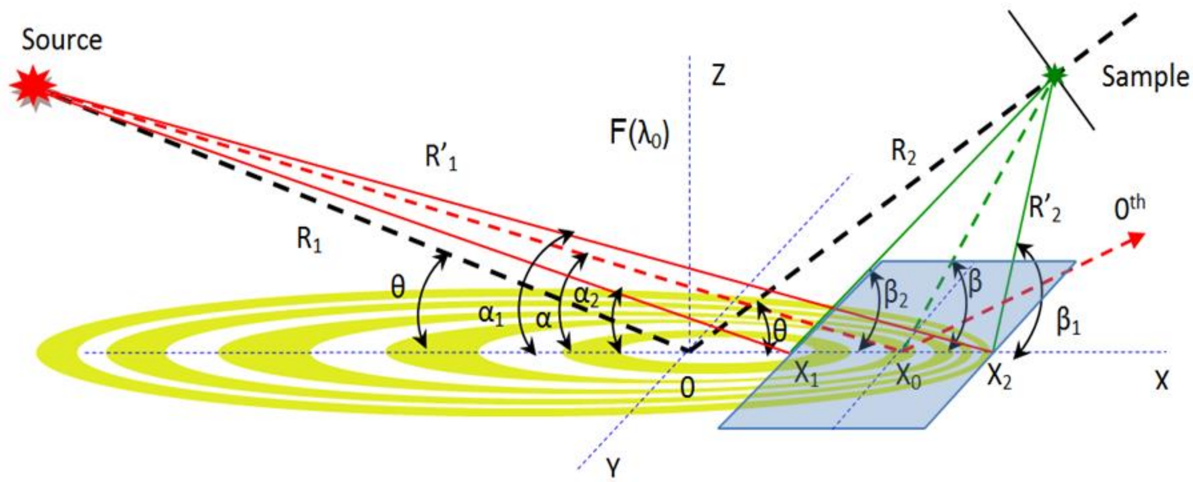

Figure 1. Layout of an off-axis reflection zone plate (RZP). The parameters associated with the figure labels are discussed within the main body of the text.

As depicted in figure 1, only an off-axis section (marked by the blue frame) of the full elliptical RZP structure is made use of in the off-axis RZP scheme. Recalling the fact that a zone plate is a hologram of a point source, the radiation from the utilized RZP section is focused along the optical axis too but with high angular dispersion due to the high off-center average line density. In addition, the specular reflection (zero order of the grating) is readily separated from the diffracted components using an off-axis scheme and, for monochromatization, a slit in a plane perpendicular to the optical axis can be utilized for energy selection. The following calculation is based on the fact that each infinitesimal part of the RZP has to fulfill the grating equation.

The input beam as emitted by the source, *S,* is dispersed along the optical axis. The same phenomenon is used in variable line-spacing (VLS) gratings in one dimension [10] to provide spectral dispersion of the input radiation.

The focal distance, $F(\lambda)$, along the axis depends on the radiation wavelength according to:

$$F(\lambda) = F(\lambda_0)\frac{\lambda_0}{\lambda} \quad (1)$$

where $F(\lambda_0)$ is the focal distance at the design wavelength, $\lambda_{0,}$ of the RZP.

All other energies are dispersed along the focal plane and are rejected by the slit. Considering an infinitesimal part of the RZP, the linear angular dispersion in the plane perpendicular to the optical axis can be locally calculated using the regular grating formula:

$$d(\cos\alpha - \cos\beta) = n\lambda \quad (2)$$

where *d* is the grating period, α is the input grazing angle on the RZP (off-axis) area, β is the diffraction angle (figure 1) and n is the number of illuminated grating lines. The X_1 and X_2 coordinates indicate margins of the working area in figure 1. Corresponding diffraction angles (β_1, β, β_2) and input angles (α_1, α, α_2) as well as the corresponding grating periods can be obtained using equation (3). After a simple transformation, we can derive the angular dispersion of the grating in the middle of the working area (X_0):

$$\Delta\beta = \frac{\Delta\lambda}{d\ \sin\beta} \quad (3)$$

where Δλ is the wavelength change with β. Taking the angular dispersion available from equation (4) and considering a pixel size of Δh at the focal plane, one can calculate the wavelength resolution as:

$$\frac{\lambda}{\Delta\lambda} = \frac{R_2'\lambda}{\Delta hd\ \sin\beta} \quad if\ \Delta h \geq \Delta S\frac{R_2'}{R_1'} \quad (4)$$

The energy dispersion, D, in the focal plane can be readily derived from (5) and expressed as:

$$D = \frac{\Delta E}{\Delta h} = \frac{E^2 d\ \sin\beta}{R_2' hc} \quad (5)$$

where Δh is the pixel size, R'_1 and R'_2 are the corresponding source–grating and grating-focus distances, respectively, and ΔS is the vertical size of the source.

Equations (5) and (6) suggest that the desired design resolution of a RZP can be tailored either by the value of β, by the period of the structure, d, and/or by the slit size Δh. Equation (5) indicates that the product of the local grating period and the sine of the diffraction angle is constant for a defined geometry and a selected energy resolution of the experiment. By combining equation (3, 4) and (5) it is now possible to calculate the local period of the optical element that satisfies the requested focusing condition along the 2D structure:

$$d = \frac{\lambda}{\sin\alpha}\left[\sqrt{1+\cot^2\alpha+\left(\frac{R_2'}{\Delta h}\,\frac{\Delta\lambda}{\lambda}\right)^2} - \, n\,\cot\alpha\right] \tag{6}$$

Using equation (7,8) it is possible to determine the missing design parameters of the RZP – the angle between the optical axis and the surface, θ, as well as the distances R_1 and R_2 between the center of the RZP and the source and the image, respectively (see figure 1):

$$\theta = \tan^{-1}\left[\frac{R_1'\ \sin\alpha + \ R_2'\ \sin\beta}{R_1'\ \cos\alpha + R_2'\ \cos\beta}\right] \tag{7}$$

$$R_1 = \frac{R_1'\ \sin\alpha}{\sin\theta} \quad and \quad R_2 = \frac{R_2'\sin\beta}{\sin\theta} \tag{8}$$

Once these parameters are determined, the RZP design is established for a specific focal length, F, at the angles α and β.

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[1]. B. Niemann, Offenlegungsschrift DE 19542679A1, (1997)
[2]. M. Brzhezinskaya, A. Firsov, K. Holldack, T. Kachel, R. Mitzner, N. Pontius, J.-S. Schmidt, M. Sperling, C. Stamm, A. Föhlisch, A. Erko, J. Synchrotron Rad., 20, 522-530, (2013).